Indian J. Phys. **51A**, 58-62 (1977)

The effect of anisotropic scattering of electrons on the galvanomagnetic coefficient in semiconductors

E. F. ELWAHEIDY

Physics Department, Faculty of Science, University of Alexandria, Alexandria, Egypt

(*Received* 13 *April* 1976)

Using a phenomenological model for the elastic and anisotropic scattering of electrons, the magneto-resistance coefficient in a semiconductor acted on by a weak magnetic field is calculated.

1. Introduction

Anisotropic galvanomagnetic effects in Bi_2Te_3 crystals have been discovered by Drabble (1958). Later, Baranskii & Korila (1964) measured the anisotropic Hall effect in *n*-Ge. The fact that anisotropic imperfections can arise in semiconductors during their irradiation by fast neutrons has been also realised experimentally by Vodopianov & Kurdianin (1965). It was therefore of particular interest to calculate the kinetic coefficients in semiconductors when anisotropic scattering of carriers is considered. Unfortunately, the lack of a complete description of the anisotropic imperfections makes the calculation very complicated. The purpose of the present work is to present the results of a simple phenomenological model analysis for the anisotropy in the collision integral of the kinetic equation.

2. Formulation of the Problem

It is well known that at low electric fields E, the distribution function of electrons can be written as,

$$f(\bar{p}) = f_0(\epsilon) + f_1(\bar{p}), \quad f_1(\bar{p}) \ll f_0(\epsilon)$$

where $f_0(\epsilon)$ is the symmetric part of the distribution and $f_1(\bar{p})$ is the asymmetric one. Let us consider the direction collinear to the unit vector $\bar{q}(q^2 = 1)$ to be distinguished regarding to the anisotropic scattering processes. Then, when the constant energy surfaces are spherical, a relaxation time exists and the kinetic equation can be written in the τ-approximation as :

$$\frac{e}{m}(\bar{E}.\bar{P}_r)\frac{df_0(\epsilon)}{d\epsilon} + \frac{e}{mc}[\bar{P}\times\bar{H}]\bar{\nabla}_p f_1(\bar{p}) = \frac{f_1(\bar{p})}{\tau(\epsilon)} + \frac{a(\epsilon)}{\tau(\epsilon)}\frac{(\bar{p}\bar{q})}{\bar{p}^2}\int d\Omega_{\bar{p}'}(\bar{p}'\bar{q})f_1(\bar{p}') \quad \ldots \quad (1)$$

where ϵ is the energy of electron ($\epsilon = p^2/2m$), m its effective mass, c is the speed of light, H is the applied magnetic field and $a(\epsilon)$ is a dimensionless parameter, characterising the anisotropic scattering degree. The form of the integral in (1) is chosen in such a way that, the properties in the $\bar{q}$ and $-\bar{q}$ directions are the same in the absence of external fields.

We seek solutions for eq. (1) of the form

$$f_1(\bar{p}) = \frac{e\tau(\epsilon)}{m}\frac{df_0(\epsilon)}{d\epsilon} P_i U_{ij} E_j. \qquad \ldots \quad (2)$$

Inserting (2) in (1) on gets,

$$\delta_{ij} + \sum_{k,l=1}^{3} e_{ikl} h_k \tau_{lj} U_{ij} = U_{ij} + a(\epsilon) q_i q_l U_{lj} \qquad \ldots \quad (3)$$

where δ_{ij} is the Kronecker delta notation, e_{ikl} is the unit antisymmetrical tensor and $\bar{h} = e\frac{\bar{H}}{mc}$.

The tensor U_{ij} has to be found in the form of linear combinations of independent tensors of the second order, which are composed of the vector components q_i and h_j. The solution of eq. (3), satisfying the Onsager's relation

$$U_{ij}(h) = U_{ji}(-\bar{h})$$

is found in the form,

$$U_{ij} = \frac{1}{1+a(\epsilon)+h^2\tau^2(\epsilon)+a(\epsilon)W^2\tau^2(\epsilon)} \times$$

$$\times [(1+a(\epsilon)\delta_{ij} - a(\epsilon) q_i q_j - \sum_{k=1}^{3} (1+a(\epsilon)\epsilon_{ijk} h_k \tau - a(\epsilon)\tau(q_i r_j - r_i q_j) + \tau^2(\epsilon) h_i h_j] \qquad \ldots \quad (4)$$

where

$$r_i = \sum_{j,k=1}^{3} \epsilon_{ijk} q_j h_k \quad \text{and} \quad W = (\bar{q}\bar{h}).$$

The conductivity tensor $\sigma_{ij}(\bar{h})$, as given by the standard methods using expressions (2) and (4), can be written in the form,

$$\sigma_{ij}(h) = (T_1 + A_1)\delta_{ij} - A_1 q_i q_j - \sum_{k=1}^{3} (T_2 + A_2)\epsilon_{ijk} h_k - A_2(q_i r_j - r_i q_j) + T_3 h_i h_j \quad \ldots \quad (5)$$

where the following notations are used,

$$T_n = \left\langle \frac{\tau^n(\epsilon)}{1+a(\epsilon)+h^2\tau^2(\epsilon)+a(\epsilon)W^2\tau^2(\epsilon)} \right\rangle$$

$$A_n = \left\langle \frac{a(\epsilon)\tau^n(\epsilon)}{1+a(\epsilon)+h^2\tau^2(\epsilon)+a(\epsilon)W^2\tau^2(\epsilon)} \right\rangle \text{ and}$$

$$\sigma_{xx} = -\frac{e^2}{3m^2}\frac{2}{(2\pi\hbar)^3}\int d\bar{p}\, p^2 \frac{df_0(\epsilon)}{d\epsilon} B$$

For weak magnetic fields $\omega\tau = h \ll 1$, we have:

$$T_n \approx \left\langle \frac{\tau^n(\epsilon)}{1+a(\epsilon)} \right\rangle - h^2 \left\langle \frac{\tau^{n+2}(\epsilon)\mu}{1+a(\epsilon)} \right\rangle,$$

$$A_n \approx \left\langle \frac{\tau^n(\epsilon)a(\epsilon)}{1+a(\epsilon)} \right\rangle - h^2 \left\langle \frac{a(\epsilon)\tau^{n+2}(\epsilon)\mu}{1+a(\epsilon)} \right\rangle$$

where

$$\mu = \frac{1+a(\epsilon)\cos\theta}{1+a(\epsilon)}$$

and θ is the angle between $\bar{q}$ and $\bar{h}$.

The resistivity tensor ρ_{jl} can be deduced from expression (5) by the relation,

$$\rho_{jl} = \frac{M_{lj}}{\Delta} \qquad \ldots (6)$$

where $\Delta = \text{Det.}|\sigma_{lj}|$ and M_{jl} is the minor of Det.$|\sigma_{lj}|$.

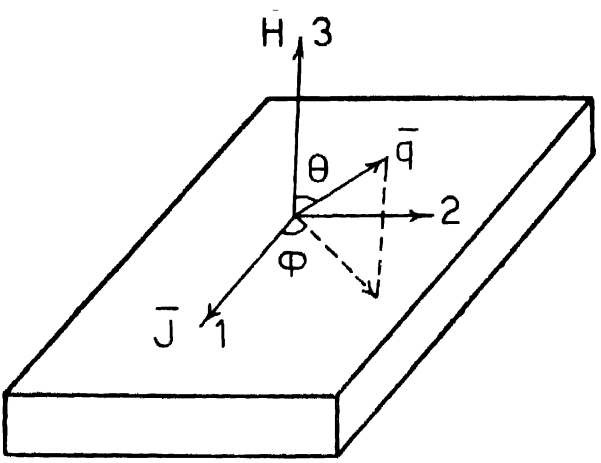

Fig. 1. The commonly used geometry

The magnetoresistance coefficient given from expressions (5) and (6), for the commonly used geometry shown in figure 1, takes the form

$$\frac{\rho(h)-\rho(0)}{h^2\rho(0)} = \frac{\Delta\rho}{h^2\rho(0)} = \frac{1}{1+\sin^2\theta\cos^2\left(\dfrac{\langle\tau\rangle}{\left\langle\dfrac{\tau}{1+a}\right\rangle}-1\right)}$$

$$\times\left[\sin^2\theta\left(\frac{\left\langle\dfrac{\tau^3}{1+a}\right\rangle}{\langle\tau^2\rangle}-\frac{\left\langle\dfrac{\tau^2}{1+a}\right\rangle^2}{\langle\tau\rangle\left\langle\dfrac{\tau}{1+a}\right\rangle}\right)+\cos^2\theta\left(\frac{\langle\tau^3\rangle}{\langle\tau\rangle}-\frac{\langle\tau^2\rangle^2}{\langle\tau\rangle^2}\right)\right.$$

$$+\sin^2\theta\cos^2\phi\left\{\sin^2\theta\left(\frac{\langle\tau\rangle\left\langle\dfrac{\tau^3}{1+a}\right\rangle-\left\langle\dfrac{\tau^2}{1+a}\right\rangle^2}{\left\langle\dfrac{\tau^2}{1+a}\right\rangle^2}-\frac{\left\langle\dfrac{\tau^3}{1+a}\right\rangle}{\langle\tau\rangle}\right.\right.$$

$$\left.\left.-\frac{\left\langle\dfrac{\tau^2}{1+a}\right\rangle}{\langle\tau\rangle\left\langle\dfrac{\tau}{1+a}\right\rangle}\right)+\cos^2\theta\left(\frac{\left\langle\dfrac{\tau^3}{1+a}\right\rangle}{\langle\tau\rangle}-\frac{\langle\tau^3\rangle}{\langle\tau\rangle}-\frac{\langle\tau^2\rangle\left\langle\dfrac{\tau^2}{1+a}\right\rangle}{\langle\tau\rangle\left\langle\dfrac{\tau}{1+a}\right\rangle}+\frac{\langle\tau^2\rangle^2}{\langle\tau\rangle^2}\right)\left.\right\}\left.\right] \quad (7)$$

When the anisotropy is in the direction 2 in figure 1 ($\theta=\phi=\pi/2$) and we have,

$$\frac{\Delta\rho}{h^2\rho(0)} = \left[\frac{\left\langle\dfrac{\tau^3}{1+a}\right\rangle}{\langle\tau\rangle}-\frac{\left\langle\dfrac{\tau^2}{1+a}\right\rangle^2}{\langle\tau\rangle\left\langle\dfrac{\tau}{1+a}\right\rangle}\right] \quad \ldots (8)$$

when the anisotropy is in the direction 3 in figure 1 ($\theta=\phi=0$), the magnetoresistance coefficient tends to its standard value in the absence of anisotropy (Kireev 1975).

$$\frac{\Delta\rho}{h^2\rho(0)} = \left[\frac{\langle\tau^3\rangle}{\langle\tau\rangle}-\frac{\langle\tau^2\rangle^2}{\langle\tau\rangle^2}\right]. \quad \ldots (9)$$

3. Discussion

It is clear from the results that, even if a simple phenomenological model is considered, the anisotropy appears in the galvanomagnetic coefficients by a relatively complicated way through a chain of averaged functions of the anisotropy degree $a(\varepsilon)$ and the relaxation time $\tau(\varepsilon)$. It is shown also that the depend-

ence of magnetoresistance coefficient on the anisotropy degree changes from one geometry to another. It should be marked that, when the magnetic field is oriented in the direction of the anisotropy, the magnetoresistance coefficient tends to its value in absence of anisotropy.

References

Baranskii P. I. & Korilo P. M. 1964 *Soviet Phys. Solid State*, **6**, 54.
Drabble J. R. 1958 *Proc. Phys. Soc.* **72**, 380.
Kireev P. S. 1975 *Semicond. Phys.* (Translated from Russian).
Vodopianov L. K. & Kurdiamn N. N. 1965 *Soviet Phys. Solid State*, **7**, 2749.